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Determining See-Saw Parameters from Weak Scale Measurements?

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Abstract

The see-saw mechanism is a very attractive explanation for small neutrino masses. However, many free parameters are introduced at a high energy scale, so analyzing other consequences of this new physics is difficult without additional hypotheses, such as flavour symmetries. In this letter we show that it is possible to parametrize the high energy physics just in terms of quantities measurable (in principle) at low energies: the neutrino Yukawa matrix, $\mathbf{Y}_\nu$, and the right-handed Majorana mass matrix, $\mathcal{M}$, can be calculated from the light neutrino masses, the MNS matrix, and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, which enters into the left-handed slepton radiative corrections. This raises the interesting possibility of obtaining model-independent information about consequences of the see-saw mechanism, such as leptogenesis. Future experiments at neutrino factories and the LHC will be crucial to further constrain these low energy quantities and test the supersymmetric see-saw scenario.

1 Introduction and notation

The observed atmospheric [1–3] and solar [4] neutrino deficits suggest that neutrinos have small but non-zero masses. These can be elegantly explained via the see-saw [5] mechanism, where the left-handed neutrinos of the Standard Model get a small mass from their small mixing with the heavy right-handed singlet neutrinos. Unfortunately, the new physics required is at a very high scale, and not directly accessible to experiments. This seems to exclude the possibility of determining the high energy theory. However, we will show in this letter that all the see-saw parameters can be determined in a unique way from some matrices that can be measured (in principle) at low energies.

We will restrict ourselves to the supersymmetric see-saw for two reasons: first, supersymmetry stabilizes the Higgs mass against the dangerous quadratic divergences that appear due to the presence of heavy particles (and in the context of the see-saw mechanism, we know that there are at least three heavy particles, namely, the right-handed neutrinos). Second, the presence of sleptons in the spectrum of the theory

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will turn out to be very useful in our task, as will be discussed later, since some of our low energy inputs are more easily determined experimentally if the theory is supersymmetric.

The leptonic part of the superpotential reads

$$W_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H_1 + {\nu_R^c}^T \mathbf{Y}_\nu L \cdot H_2 - \frac{1}{2} {\nu_R^c}^T \mathcal{M} \nu_R^c, \tag{1}$$

where L_i and e_{Ri} $(i = e, \mu, \tau)$ are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and H_1 (H_2) is the hypercharge $-1/2$ $(+1/2)$ Higgs doublet. $\mathbf{Y_e}$ and $\mathbf{Y}_\nu$ are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and $\mathcal{M}$ is a 3×3 Majorana mass matrix that does not break the SM gauge symmetry. We do not make any assumptions about the structure of the matrices in eq.(1), but simply consider the most general case. Then, it can be proved that the number of independent physical parameters is 21: 15 real parameters and 6 complex phases [6].

It is natural to assume that the overall scale of $\mathcal{M}$, denoted by M, is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian contains a Majorana mass term for the left-handed neutrinos:

$$\delta\mathcal{L}_{lep} = {e_R^c}^T \mathbf{Y_e} L \cdot H_1 - \frac{1}{2} \nu^T \mathcal{M}_\nu \nu + \text{h.c.}, \tag{2}$$

with

$$\mathcal{M}_\nu = \mathbf{m_D}^T \mathcal{M}^{-1} \mathbf{m_D} = {\mathbf{Y}_\nu}^T \mathcal{M}^{-1} \mathbf{Y}_\nu \langle H_2^0 \rangle^2, \tag{3}$$

suppressed with respect to the typical fermion masses by the inverse power of the large scale M. In what follows, it will be convenient to extract the Higgs VEV by defining

$$\kappa = \mathcal{M}_\nu / \langle H_2^0 \rangle^2 = {\mathbf{Y}_\nu}^T \mathcal{M}^{-1} \mathbf{Y}_\nu, \tag{4}$$

where $\langle H_2^0 \rangle^2 = v_2^2 = v^2 \sin^2\beta$ and $v = 174$ GeV. Working in the flavour basis in which the charged-lepton Yukawa matrix, $\mathbf{Y_e}$, and the gauge interactions are flavour-diagonal, the κ matrix can be diagonalized by the MNS [7] matrix U according to

$$U^T \kappa U = \text{diag}(\kappa_1, \kappa_2, \kappa_3) \equiv D_\kappa, \tag{5}$$

where U is a unitary matrix that relates flavour to mass eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{6}$$

and the κ_i can be chosen real and positive. U can be written as

$$U = V \cdot \text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1) \quad , \tag{7}$$

where ϕ and ϕ' are CP violating phases (if different from 0 or π) and V has the ordinary form of the CKM matrix

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \qquad (8)$$

The κ matrix, eq.(4), is at the moment our only experimental hint about the high energy physics that generates the neutrino masses. Unfortunately, it is not enough to reconstruct the whole theory. However, there is a second window onto the high energy physics apart from the neutrino mass matrix: radiative corrections. Between the GUT scale and the Majorana mass scale, M, neutrino Yukawa couplings affect the renormalization of certain parameters of the theory through the combination $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. These contributions can leave some signatures at low energies and thus provide additional information about the theory at high energies. Remarkably enough, the information provided by radiative corrections is complementary to that provided by κ and permits the reconstruction of *the complete* high energy theory.

2 General procedure

We take as our weak scale inputs the neutrino mass matrix, related to κ, and

$$P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu \quad . \qquad (9)$$

At this stage, we do not have much information about P and we prefer to interpret it simply as a way to parametrize our ignorance of the high energy physics. However, this parametrization will turn out to be very convenient, because P has a very precise physical meaning.

Now we turn to the determination of $\mathbf{Y}_\nu$ and $\mathcal{M}$ from κ and P. We can always choose to work in the basis where the charged lepton mass matrix is diagonal. We can also choose to work in a basis of right-handed neutrinos where $\mathcal{M}$ is diagonal

$$\mathcal{M} = \mathrm{diag}(\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3) \equiv D_\mathcal{M}, \qquad (10)$$

with $\mathcal{M}_i \geq 0$. In this basis, the neutrino Yukawa matrix must be necessarily non-diagonal, but can always be diagonalized by two unitary transformations:

$$\mathbf{Y}_\nu = V_R^\dagger D_Y V_L. \qquad (11)$$

V_L and D_Y can be determined from P, since, using eq. (11),

$$P \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = V_L^\dagger D_Y^2 V_L. \qquad (12)$$

On the other hand, from $\kappa = \mathbf{Y}_\nu^T D_\mathcal{M}^{-1} \mathbf{Y}_\nu$ and eq. (11),

$$D_Y^{-1} V_L^* \kappa V_L^\dagger D_Y^{-1} = V_R^* D_\mathcal{M}^{-1} V_R^\dagger, \qquad (13)$$

where the left hand side of this equation is known (κ is one of our inputs, and V_L and D_Y were obtained from eq. (12)). Therefore, V_R and $D_\mathcal{M}$ can also be determined. This shows that, working in the basis where the charged lepton Yukawa coupling, $\mathbf{Y_e}$, the right-handed Majorana mass matrix, $\mathcal{M}$, and the gauge interactions are all diagonal, it is possible to determine *uniquely* the heavy Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa coupling, $\mathbf{Y}_\nu = V_R^\dagger D_Y V_L$, starting from κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. Notice that the see-saw formula, eq. (4), is only valid at the Majorana mass scale and not at low energies, so all the parameters in eqs.(12) and (13) should be understood at M. Therefore, the observed κ should be run from the electroweak scale to the Majorana mass scale with the corresponding Renormalization Group Equations (RGEs) [1].

At this point it is worth checking that the number of physical parameters is identical at high and low energies. If we want to reconstruct the whole theory from low energy data, we must have 15 real parameters and 6 complex phases at low energies, and this is actually the case. In the basis we have chosen to work in, κ contains six real parameters and three complex phases, $\mathbf{Y_e}$ is determined by three real parameters, and P contains six real parameters and three complex phases, which add up to 15 real parameters and 6 complex phases. It is not immediately obvious that they are all independent, but we have proved that indeed they are, calculating $\mathbf{Y}_\nu$ and $\mathcal{M}$ explicitly from κ and P.

Reconstructing $\mathcal{M}$ and $\mathbf{Y}_\nu$ from κ and P is appealing both for theoretical and phenomenological reasons. κ and P contain the same information about the see-saw as $\mathcal{M}$ and $\mathbf{Y}_\nu$, but κ is expressed in terms of observable neutrino masses, mixing angles and phases, and P can in principle be extracted from renormalization group effects. It is therefore straightforward to restrict κ and P matrix elements to lie within their experimentally allowed ranges. The experimentally allowed $\mathcal{M}$ and $\mathbf{Y}_\nu$ can then be reconstructed. These could be applied to the study of the cosmological baryon asymmetry generated from the out-of-equilibrium decay of the ν_Rs [8]: for certain choices of $\mathbf{Y}_\nu$ and $\mathcal{M}$, a sufficiently large lepton asymmetry is produced in the decay of the ν_Rs, and subsequently reprocessed into a baryon asymmetry by non-perturbative Standard Model B+L violating processes. Starting from κ and P, we can readily calculate the implications of experimental measurements for the leptogenesis scenario, and study the dependence of the CP violating asymmetry in ν_R decay [2] on weak scale masses and couplings [9].

It is interesting to compare this result to the quarks, where the u_R and the u_L share a Dirac mass—there is no undetermined GUT-scale mass. The up quark masses squared are proportional to the eigenvalues of $\mathbf{Y}_u^\dagger \mathbf{Y}_u$, which is also the combination that appears in the RGEs. This is in contrast to the neutrino sector, where the light neutrino mass matrix is $\kappa = \mathbf{Y}_\nu^T \mathcal{M}^{-1} \mathbf{Y}_\nu$, and the left-handed slepton RGEs depend on $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$. So, the sleptons provide additional information, complementary to the lepton mass matrices. In the neutrino sector, κ, rather than $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, is directly measurable at the weak scale. $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ contributes to the renormalization group running from the high energy scale M_X to $\mathcal{M}_i$; indeed, it is the high scale input which (in conjunction with κ) allows us to determine $\mathcal{M}$.

[1] This seems to require knowing the Majorana mass scale from the beginning, however, in a numerical calculation, M can be computed recursively.

[2] The generated lepton asymmetry also depends on cosmological parameters, so additional assumptions are required to calculate the baryon asymmetry.

3 Present and future constraints on κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$

All the matrix elements of κ are in principle measurable, because they are determined from the light neutrino masses and the angles and phases of the MNS matrix. Three masses, three mixing angles and three phases are required to fully reconstruct κ. Atmospheric neutrino data determine the neutrino mass difference $|m_3^2 - m_2^2|$, and the mixing angle θ_{23}. Solar data allow various values for $m_2^2 - m_1^2$ and θ_{12}, although present data seem to favour the large angle MSW solution [10]. Also, reactor experiments [11] constrain θ_{13} to be small. Upcoming and proposed experiments hope to determine the solar solution, and measure the angle θ_{13}, the sign of $m_3^2 - m_2^2$, and the CP violating phase δ [12]. The overall scale of the neutrino masses could be determined from microwave background and large scale structure observations if $\sum_i m_i \gtrsim .1$ eV [13]—alternatively one could take the largest mass to be the largest mass difference. The two remaining "Majorana" phases ϕ and ϕ' contribute to $\Delta L = 2$ neutrino interactions, such as neutrinoless double beta decay [14], which could measure a combination of ϕ and ϕ'. An additional $\Delta L = 2$ process would have to be measured to determine ϕ and ϕ' separately.

We now turn to constraining P, which appears in the RGE of the left-handed slepton soft mass matrix, $\mathbf{m}_L^2$. Since P is related to radiative corrections, it is expected that the absolute value of any of its elements is $\lesssim 16\pi^2$. To further constrain P, it is necessary to make an ansatz about the high energy physics, and in what follows it will be assumed that the slepton soft mass matrices at high energies are proportional to the identity. Of course, there is no theoretical reason to impose this condition, but it is naturally obtained in some well motivated scenarios, for instance, minimal supergravity, dilaton-dominated SUSY breaking or gauge-mediated SUSY breaking. Also, model independent analyses of flavour changing processes suggest approximate universality of the soft terms. In making this assumption, we are imposing some hypotheses on the high energy theory and thus our approach is not strictly bottom-up. However, we find the assumption of perfect universality very natural (it is certainly the most popular one) and a very small price to pay for determining all the see-saw parameters from low energy data. Under this assumption, the low energy left-handed slepton mass matrices read, in the leading-log approximation,

$$\left(m^2_{\tilde{\ell},\tilde{\nu}}\right)_{ij} \simeq (\text{diagonal part})_{\tilde{\ell},\tilde{\nu}} - \frac{1}{8\pi^2}(3m_0^2 + A_0^2)(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{ij} \log \frac{M_X}{M} , \qquad (14)$$

where "diagonal-part" includes the tree level soft mass matrix, the radiative corrections from gauge and charged lepton Yukawa interactions, and the mass contributions from F- and D-terms (that are different for charged sleptons and sneutrinos). The off-diagonal elements in eq. (14) induce rare lepton flavour violating processes, such as $\mu \to e\gamma$. The present upper bounds on their branching ratios can be used to constrain the absolute values of the off-diagonal elements of $P = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, especially $|(\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)_{12}|$, up to a log dependence on the right-handed Majorana masses. For example, model independent analyses obtain $(m_L^2)_{12} \lesssim 20\,\text{GeV}^2$ for a slepton mass of $100\,\text{GeV}$ [15], which translates into $|P_{12}| \lesssim 6 \times 10^{-3}$. The observation of lepton flavour violating processes would set a lower bound on $|P_{ij}|$, $i \neq j$, for a certain choice of supersymmetric parameters.

Sparticle production at future colliders offers the possibility of further constraining the left-handed slepton mass matrices and therefore the moduli and phases of the elements of P. The production of sneutrinos and the observation of their decays could determine their masses and couplings to the charged leptons, which would constrain the diagonal elements of m_L^2 (these can also be constrained from the sneutrino masses and the $\ell_i \to \ell_j \gamma$ branching ratios). The diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ contribute to the running of the diagonal elements of the left-handed slepton soft mass matrix, as do the gauge and charged lepton Yukawa couplings. So, elements P_{ii} that are of order the gauge couplings could be determined from the sneutrino masses. For a hierarchical $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, this would give $[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]_{33}$, and possibly $[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]_{22}$. It might be possible to determine $[\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu]_{11}$ from accurately measured sneutrino mass differences.

The matrices $\mathbf{m}^2_{\tilde{\ell},\tilde{\nu}}$ are hermitian, so their off-diagonal elements are complex. Naively, from sneutrino oscillation experiments one could hope to extract the three phases of $\mathbf{m}^2_{\tilde{\nu}}$, however, it is likely that only one combination is measurable. In these experiments, a flavour eigenstate sneutrino could be produced with a lepton of one flavour (say e^+), oscillate among the different $\tilde{\nu}$ mass eigenstates, and then decay into a lepton of another flavour (*e.g.* μ^-). As discussed in [16], this process only involves one phase, which could be measured (for sneutrino mass differences of order the decay rate) in the asymmetry between the observed number of $e^+\mu^-$ and $e^-\mu^+$. The reason why there is only one phase is that the lepton number conserving [3] sneutrino mass matrix $[\mathbf{m}^2_{\tilde{\nu}}]_{ij}$ can be diagonalized, in the basis defined after eq. (4), by $W \mathbf{m}^2_{\tilde{\nu}} W^\dagger = D_{\mathbf{m}^2}$. The unitary matrix W has six phases, five of which can be rotated away by choosing the relative phases of the charged leptons and sneutrinos. This removal of phases is similar to that in the CKM matrix. It is possible because the phases of the left-handed leptons are not fixed by making their mass matrix diagonal and real, so their two relative phases can be chosen to remove two phases, either in W *or* in the MNS matrix. In the case of sneutrino oscillations, two of the relative phases of the left-handed leptons used to redefine W will reappear in the MNS matrix, but this does not have any physical effect since this experiment does not involve neutrinos. Thus, to measure the remaining two phases we need an experiment probing the sneutrino-neutrino-neutralino vertex, that depends on both W and the MNS matrix. Experiments that involve the MNS matrix are difficult to perform, so measuring these two phases does not seem feasible. An alternative approach might be to look for CP violation in sneutrino-anti-sneutrino oscillations [17]. Sneutrinos can oscillate into anti-sneutrinos due to small $\Delta L = 2$ sneutrino masses $[m^2_{\tilde{\nu}\tilde{\nu}}]_{ij} \tilde{\nu}_i \tilde{\nu}_j + h.c.$, which are the soft SUSY breaking analogy of the neutrino Majorana masses: $[m^2_{\tilde{\nu}\tilde{\nu}}]_{ij} \sim v_2^2 m_0 \kappa_{ij}$. However, this process is only observable in a small area of the SUSY parameter space [17], so it is unlikely that the phases could be extracted from these phenomena.

4 Discussion

In the best of all supersymmetric worlds, where soft terms are universal at the GUT scale, and all masses, couplings and phases have been measured at the weak scale,

[3] we neglect for the moment the sneutrino "Majorana" mass matrix, appearing in $[m^2_{\tilde{\nu}\tilde{\nu}}]_{ij} \tilde{\nu}_i \tilde{\nu}_j + h.c.$

it is possible to calculate the GUT-scale inputs of the SUSY see-saw from measured quantities. However, in practice it would be very difficult to make all the weak scale measurements required to determine the neutrino Yukawa matrix, $\mathbf{Y}_\nu$, and the right-handed neutrino Majorana mass matrix, $\mathcal{M}$. The moduli of the off-diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ induce lepton flavour violation in the left-handed slepton mass matrices, so could be measurable. The diagonal elements of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ are more difficult: they modify the diagonal elements of the left-handed slepton soft mass matrix, so they would be hard to disentangle from gauge and charged lepton Yukawa contributions to the RG running. Finally, most of the CP violating phases in $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ and the MNS matrix seem practically unattainable.

A less ambitious, and at this stage more practical, interpretation of our results, is that it is possible to describe the see-saw parameter space using just inputs at the electroweak scale, which have a very straightforward physical interpretation (neutrino masses and mixings, rates for rare flavour changing processes, sneutrino masses...). Since we make our experiments at the electroweak scale, it is obviously much more natural to use a parameter space spanned by quantities measurable at the electroweak scale rather than at the GUT scale. This approach has the immediate consequence that the see-saw parameter space is already constrained by neutrino data and rare lepton decays, and might be further constrained in the future with the advent of neutrino factories and LHC. Therefore, model independent conclusions about the seesaw mechanism can be drawn more readily in this bottom-up formulation than in a top-down approach, where a priori the parameter space is not constrained at all! [4]

Our parametrization is very convenient to look for ways to test the see-saw mechanism. We have shown that it is difficult to rule out the see-saw mechanism from neutrino data and radiative effects on left-handed slepton soft mass matrices: for *any* combination of low energy neutrino parameters (encoded in κ) and left-handed slepton mass matrices (that can be parametrized by P, as in eq.(14)), it is *always* possible to find a neutrino Yukawa coupling and a Majorana mass matrix that reproduce those low energy observables [5]. If those $\mathbf{Y}_\nu$ and $\mathcal{M}$ are not compatible with perturbativity or the experimental lower bounds on Majorana masses, it would be a blow against the see-saw. Also, the observation of lepton flavour violation not encoded in κ and P (for instance from the right-handed slepton mass matrix), would be an indication for physics other than the see-saw with universal soft terms. In this parametrization, neutrino masses and mixing angles, and radiative effects on slepton masses, are inputs, and in this sense cannot be regarded as predictions of the see-saw mechanism. Therefore, to test the see-saw, we need another low energy effect which is a consequence of it, such as the baryon asymmetry of the Universe. The CP asymmetry in right-handed neutrino decay is indeed a prediction of the see-saw and, when combined with assumptions about the early evolution of the Universe, could be compared with the cosmological baryon asymmetry.

In summary, we have shown that it is possible to determine the GUT scale inputs of

[4] An alternative model independent analysis of the see-saw mechanism can be found in [18].

[5] Notice that this is not obvious in the top-down approach: for example, it is not immediately clear that there is a region of the $\mathbf{Y}_\nu$ and $\mathcal{M}$ parameter space consistent with any experimentally determined neutrino parameters and with the upper bounds on rare lepton decays.

the see-saw from parameters that are in some sense measurable at the weak scale. The right-handed neutrino Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa matrix, $\mathbf{Y}_\nu$, can be calculated from the light neutrino masses, the MNS matrix, and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ (which enters into the renormalization group equations of the left-handed slepton soft mass matrix). This does not conflict with decoupling theorems because $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ (whose indices are left-handed) is effectively a high-scale input: it contributes to the slepton mass matrix via renormalization group running at scales above M. We briefly discussed the (not very encouraging) prospects of measuring the magnitudes and phases of all κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ matrix elements, in a model with universal soft terms. A more realistic approach is to impose all the available experimental constraints on κ and $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$, vary the matrix elements over their experimentally allowed ranges, and calculate the resulting allowed values of $\mathbf{Y}_\nu$ and $\mathcal{M}$. We will pursue this bottom-up approach to the see-saw (and its application to leptogenesis) in subsequent work [9].

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